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Wave Properties of nonlocal Euler beam model in Carbon nanotubes

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Abstract— In this paper Eringen's nonlocal theory of elasticity is applied to study the wave properties of single walled carbon nanotubes. The effects of wave number and nonlocal scaling parameter (e0a) on the frequency, phase velocity and group velocity are discussed for a particular material of carbon nanotube.

Index Terms— Carbon nanotube, Euler beam theory, Wave propagation, Nonlocal elasticity.

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1 Introduction

The nonlocal theory of elasticity due to Eringen [1] is widely used in nano mechanical problems of carbon nanotubes such as crack, wave propagation and vibration analysis. This theory discuss scale effects and large range atomic interactions. The introducing of carbon nanotubes by Lijima [2] has motivated a good research in the field of nanodevices and nanocomposites. Experiments at the nanoscale are very difficult and atomistic modeling remains is expensive for large sized atomic system. The size dependent continuum models are playing an important role in the study of carbon nanotubes [3].The main reason these size dependent continuum mechanics are used is because at small length scales the material properties of microstructures [4, 5, 6] become importantly significant and the influence of the micro size cannot be ignored.

In nonlocal elasticity theory the small scale effects are captured by assuming that the stress at a point as a function not only of the strain at that point but also of the strains at all other points of the domain. This is in accordance with predictions from atomic lattice dynamics. It is important to note that the stress tensors defined in the nonlocal elasticity theory are nonlocal ones which is different from the local stress tensor defined in classical elasticity theory [7].Therefore, it should be kept in mind that in deriving a nonlocal beam model or any other nonlocal continuum model, all formulations involving stress components are based on the nonlocal stress tensor, not on the local ones [8, 9].

The main objective of this paper is to study the wave properties of single walled carbon nanotube. The related governing equations for the Euler beam model are derived. The wave and vibration properties of carbon nanotube based on nonlocal elasticity are presented. The scale effects on wave speed and group speed dispersions are studied for a particular material of carbon nanotube. The effect of nanoscale parameter on the different dispersive properties of wave propagation of the carbon nanotube is shown numerically.

2 Constituent equations of Nonlocal beam model

The nonlocal beam models in earlier literature are based on a kind of the nonlocal constitutive relations given by Eringen's [1] equations as

$$\left[1-(e_0 a)^2 \nabla^2\right] t_{kl} = \lambda \varepsilon_{rr} \delta_{kl} + \mu \varepsilon_{kl} \tag{1}$$

where t_{kl} is the nonlocal stress tensor, ε_{kl} is the strain tensor, λ, μ are materials constants, a is an characteristic length and e_0 is a constant for adjusting the model in matching certain experimental results. For this model, the size in thickness and width are must be smaller than the length size. Therefore , for the beams with transverse motion in x-y plane, the nonlocal constitutive relation (1) can be approximated to one dimensional form as

$$\left[1-(e_0 a)^2 \frac{\partial^2}{\partial x^2}\right] t_{xx} = E\varepsilon \tag{2}$$

$$\left[1-(e_0 a)^2 \frac{\partial^2}{\partial x^2}\right] t_{xy} = G\gamma \tag{3}$$

Where E is the Young's modulus, G is the shear modulus, ϵ is the axial strain and γ is the shear strain .These equations constitute nonlocal relation for the nonlocal beam model.

2.1 Equation of motion for Nonlocal Euler beam model

For the Euler beam model, the bending moment M is independent and the shear force S is related to the bending moment through

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the relation $S = \frac{\partial M}{\partial x}$. The governing equations of the nonlocal nano beam model are given by

$$\frac{\partial S}{\partial x} + P = \rho A \frac{\partial^2 V}{\partial x^2} \tag{4}$$

$$\frac{\partial M}{\partial x} + S = \rho I \frac{\partial^2 \psi}{\partial t^2} \tag{5}$$

Where P is the distributed force along X-axis , A is the area of cross section of the beam $I = \int_A y^2 dA$ is the moment of inertia, V is the transverse displacement and ψ is the rotation angle of cross section of the beam. For the nano beam model the axial strain and shear strain are given by

$$\varepsilon = \frac{\partial \psi}{\partial x} \tag{6}$$

$$\gamma = \frac{\partial V}{\partial x} - \psi \tag{7}$$

The resultant bending moment and shear force are given by

$$M = \int_A y t_{xx} dA \tag{8}$$

$$S = \int_A t_{xy} dA \tag{9}$$

These equations have been obtained by Peddieson [9] for the static problems and by Wang [6] for dynamic problems .Further the general expressions of the bending moment and shear forces for the nonlocal Euler beam model have presented in Lu et [4].Using the relation between the nonlocal bending moment M and resultant shear force S in the equation (8) we obtain

$$\frac{\partial^2 M}{\partial x^2} + P = \rho A \frac{\partial^2 V}{\partial t^2} \tag{10}$$

According to the linear theory of Euler-Bernoulli beam, the nonlocal constitutive relation (2) can be put as

$$\left[1 - (e_0 a)^2 \nabla^2\right] t_{xx} = E\varepsilon \tag{11}$$

Here ∇^2 is the Laplacian operator .The above equations can be written in the Cartesian form as

$$\left[t_{xx} - (e_0 a)^2 \frac{\partial^2 t_{xx}}{\partial x^2}\right] = Ey \frac{\partial \psi}{\partial x} \tag{12}$$

Multiplying by y on both sides of equation (17) and integrating over the cross section area A of the beam, we have

$$\int_A y t_{xx} - (e_0 a)^2 \frac{\partial^2 \int_A y t_{xx}}{\partial x^2} = E \int_A y^2 dA \frac{\partial \psi}{\partial x} \tag{13}$$

Using equations (10), we get

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = EI \frac{\partial \psi}{\partial x} \tag{14}$$

For Euler beam of one dimensional we take shear strain γ then we get $\gamma = \frac{\partial V}{\partial x}$ and the equation given by

$$M = EI \frac{\partial^2 V}{\partial x^2} + (e_0 a)^2 \left[\rho A \frac{\partial^2 V}{\partial t^2} - P\right] \tag{15}$$

Finally, the equation of motion of the nonlocal Euler beam model can be obtained by differentiating (15) twice with respect to the variable x and using earlier equation as

$$EI \frac{\partial^4 V}{\partial x^4} + \rho A \frac{\partial^2}{\partial t^2}\left(V - (e_0 a)^2 \frac{\partial^2 V}{\partial x^2}\right) = \left[P - (e_0 a)^2 \frac{\partial^2 P}{\partial x^2}\right]$$

(16) (16)

2.2 Solution and wave properties of nonlocal Euler beam model

For a harmonic wave propagation in the carbon nanotube governed by the equation of motion(16) the corresponding solution can be written in a complex form as $V = A_0 e^{i(wt - Kx)}$ by taking transverse force being zero i.e P = 0,and simplifying the equation(16) we obtain the dispersion relation as

$$c_1^2 K^4 - \omega^2 \left[1 + K^2 (e_0 a)^2\right] = 0 \tag{17}$$

Where $c_1 = \sqrt{EI/\rho A}$ .Making use of(17) we can find the angular frequency ω ,Phase velocity c_p and Group velocity c_g of the nonlocal Euler beam model as

$$\omega = \frac{c_1 K^2}{\sqrt{1+(e_0 a)^2 K^2}} \quad (18)$$

$$c_p = \frac{\omega}{K} = \frac{c_1 K}{\sqrt{1+(e_0 a)^2 K^2}} \quad (19)$$

$$c_g = \frac{\partial \omega}{\partial K} = \frac{c_1 K\left[2+K^2(e_0 a)^2\right]}{\left[1+K^2(e_0 a)^2\right]} \quad (20)$$

If can be observed that if we neglect the nonlocal (or) small Scale effect i.e. $e_0 a = 0$, we get results of classical Euler beam model [7].

3 Numerical treatment and Discussion

Based on the formulation is obtained above with the nonlocal Euler beam model and wave properties of single walled carbon nanotube is discussed here. The material properties of carbon nanorod are taken as Young modulus E = 106pa, density $\rho = 2270 \, kg/m^3$ and we used for free clamped boundary conditions are $\omega = 0$ and $\frac{\partial \omega}{\partial x} = 0$ at each end of the tube.

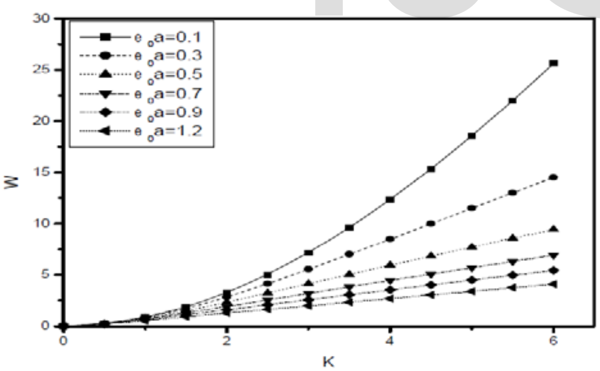

Figure 1: The effect of nonlocal scaling parameter($e_0 a$)on wave number dispersion on carbon nanotube

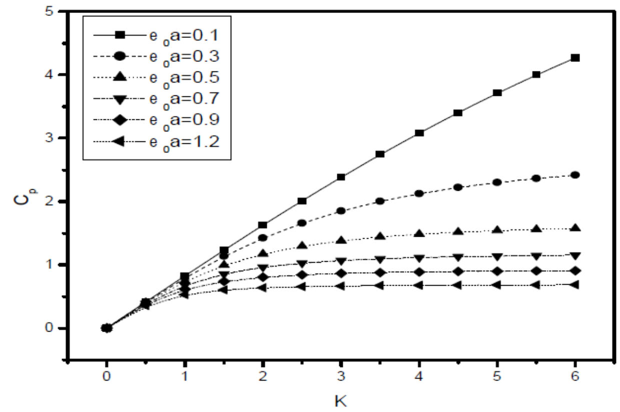

Figure 2: Effect of nonlocal scaling parameter($e_0 a$) on phase speed dispersion on carbon nanotube

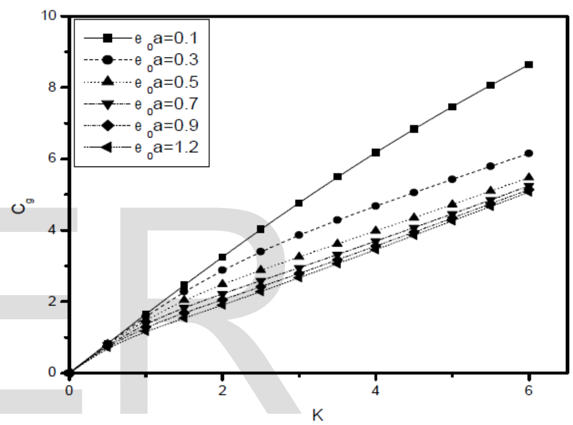

Figure 3: Effect of nonlocal scaling parameter ($e_0 a$) on group speed dispersion on carbon nanotube

Figures 1, 2 and 3 shows that the deviations of the dispersions curves based on the nonlocal beam model from those based on the local model increase with the increase of the wave numbers. It can be observed that the decreasing rates of the frequencies with $(e_0 a)$ for higher wave numbers are faster than those for smaller ones. It shows that for very small wave numbers the influence of the nonlocal effect is also very small. Therefore, in the range for small wave number the classical beam .Theories are acceptable.

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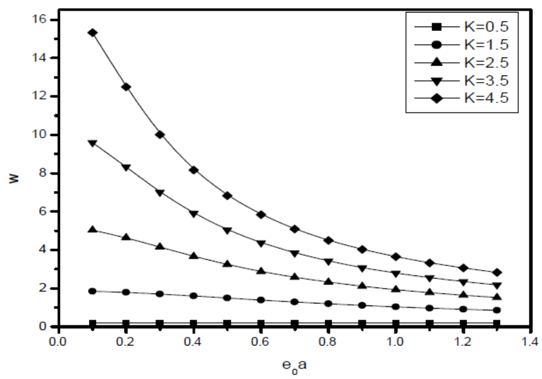

Figure 4: Effect of wave number dispersion (K) on nonlocal scaling parameter on carbon nanotube

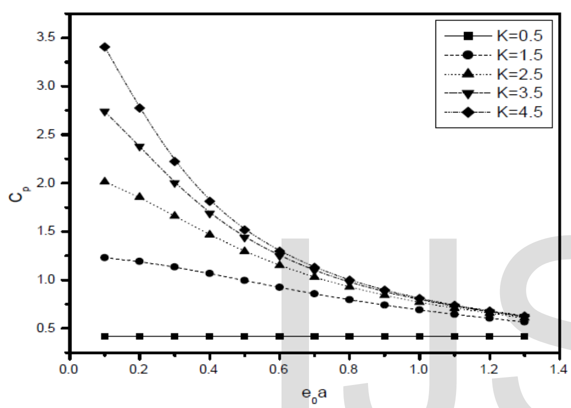

Figure 5: Effect of Phase speed dispersion on nonlocal scaling parameter on carbon nanotube

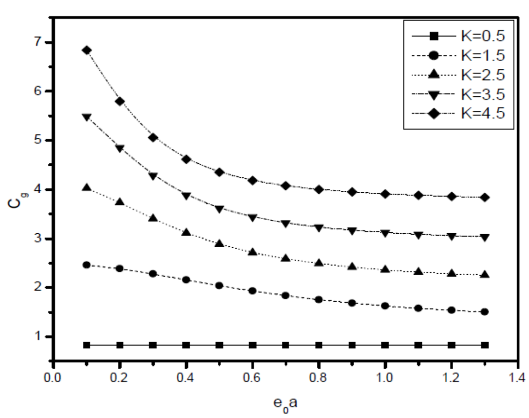

Figure 6: Effect of group speed dispersion on nonlocal scaling parameter on carbon nanotube

Figures 4 , 5 and 6 shows the changes of dispersion curses of carbon nanotube with the non-dimensional nonlocal parameter (e_0a) based on the nonlocal Euler beam model. It is seen that frequencies of phase velocity and group velocity decrease with e0a.This means that the dynamical properties of frequencies, phase and group velocities of the nanotubes based on the classical beam theories are over estimated. Since the nonlocal elasticity theory reduce to the classical elasticity theory in the long wave length limit and reduces to the atomic lattice dynamics in the short wave length limit, the effect of nonlocal parameter e0a has a significant Influence on the dynamic properties of the beam structures based on the nonlocal beam model. Therefore, the removable choice of the value of the parameter e0a is Crucial to ensure that the validity of the nonlocal model.

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